Positive Integral solution of the Lens Formula

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This short article aims to find the positive integral solution of the famous Lens Formula in the theory of light:

$$\frac{1}{f}=\frac{1}{u}+\frac{1}{v} \qquad \ldots.(1)$$

that is, f, u, v must be positive integers.

Note that (1) is equivalent to :

$$f=\frac{uv}{u+v} \qquad \ldots.(2)$$

Let p = H.C.F. (u, v).

Then u = pm, v = pn , where p is a positive integer and H.C.F. (m,n) = 1.

It follows that H.C.F. (mn, m + n) = 1.

From (2), we have

$$f=\frac{(pm)(pn)}{pm+pn}=\frac{pmn}{m+n}$$

Since f is a positive integer and H.C.F.(mn, m + n) = 1,

(m + n) divides p completely.

Therefore p = k (m + n) where k is a positive integer.

We obtain our solution:

$$f=kmn, \quad u=km(m+n), \quad v=kn(m+n) \qquad \ldots.(3)$$

where k , m, n are all positive integers.

For example, if we take k = 1, m = 2, n = 3, then by (3), f = 6, u = 10 and v = 15 ,

$$\frac{1}{6}=\frac{1}{10}+\frac{1}{15}$$

Exercise

(1) Assume f, u, v are positive integers have **no common factor** satisfying $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$,

prove that u + v is a complete square.

(2) Let f, u, v are positive integers satisfying $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$,

prove that $f^2+u^2+v^2$ is a complete square.